National Technical University of Ukraine
"Igor Sikorsky Kyiv Polytechnic Institute"

Department of Mathematical Analysis and Theory Probability

Higher Mathematics. Part 3. Series and Functions of a Complex Variable.

Work program for the credit module of the academic discipline "Higher Mathematics. Part 3. Series and Functions of a Complex Variable" (Syllabus)

Details of academic discipline

Level of higher education	First (bachelor's)
Field of knowledge	17 Electronics, Automation, and Electronic Communications
Special	172 Electronic communications and radio engineering
Educational program	**1. Intelligent technologies of radio electronics** **2. Information and Communication Radio Engineering** **3. Radio-technical computerized systems** **4. Radio-electronic warfare technologies**
Status of the discipline	Regulatory
Form of study	Full-time (day)/distance learning
Year of study, semester	2nd year, fall semester
Scope of the discipline	150 hours (54 hours – Lectures, 54 hours – Practical lessons, 42 hours – Independent work)
Semester control/control measures	Exam, MODULE TEST , Calculation work
lesson schedule	http://rozklad.kpi.ua
Language of instruction	Ukrainian
Information about the course instructor /	**Lecturer**: Candidate of Physical and Mathematical Sciences, Associate Professor of the Department of Mathematical Analysis and Probability Theory, Oleksandr Oleksandrovych Dykhovychny

teachers	a.dyx@ukr.net , mobile +38(067)9005262 **Practical**: Kateryna Kostiantivna Moskvycheva, Candidate of Physical and Mathematical Sciences, Senior Lecturer, Department of Mathematical Analysis and Probability Theory ймовірностей,baka.ni.tsukeru@gmail.com Anna Maslyuk, PhD in Physics and Mathematics, Senior Lecturer at the Department of Mathematical Analysis and Probability Theory, masliukgo@ukr.ne Tetiana Malovychko, PhD in Physics and Mathematics, Associate Professor of the Department of Mathematical Analysis and Probability Theory, tatianamtv@protonmail.com
Course placement	https://do.ipo.kpi.ua/course/view.php?id=6976

Curriculum

1. Description of the academic discipline, its purpose, subject matter, and learning outcomes

Description of the discipline	In accordance with the curriculum, the credit module "Higher Mathematics. Part 3. Series and Functions of a Complex Variable" is part of the academic discipline "Higher Mathematics" (ZO 7), belongs to the cycle of mathematical and natural science training, and is of paramount importance in the training of specialists. It is necessary for the successful mastery of special disciplines. This credit module is based on the knowledge students have acquired while studying mathematics in secondary school. The discipline "Higher Mathematics" is one of the fundamental general education disciplines that form the theoretical basis for the training of engineers and programmers. The knowledge and skills acquired by students while studying this discipline are used in the future when studying many subsequent disciplines of professional training of specialists with basic and complete higher education. While studying this discipline, students will learn: the basics of series theory; the theory of complex variable functions; integral transformations.
Objectives of the discipline	The objectives of the course are: • to develop in of education logical thinking, developing their intellect and abilities; • the formation of the necessary intuition and erudition in the application of mathematics, the cultivation of applied mathematical culture; • the formation of abilities to independently use and study literature on mathematics, the development of flexibility thinking, creative independence, and action.
Subject of the academic discipline	General mathematical properties and patterns. Series theory; theory of complex variable functions; integral transformations.

Competencies	The aim of the course is to develop students' abilities: • ability to think abstractly, analyze, and synthesize (GC 1); • ability to apply knowledge in practical situations (GC 2); • the ability to learn and master modern knowledge (PC 07); • ability to identify, pose, and solve problems (GC 08);
Program learning outcomes	Apply fundamental and applied sciences to analyze and develop processes occurring in telecommunications and radio engineering systems (PLO13)

2. **Prerequisites and post-requisites of the discipline (place in the structural-logical scheme of training under the relevant educational program)**

Post-requisites: The course "Higher Mathematics" is part of the cycle of mathematical and natural science training and is of paramount importance in the training of specialists, preceding the courses "Computer Science" and "Programmable Tools in Intelligent Radio Electronics."

3. Contents of the academic discipline

Title of sections and topics	Number of hours			
	Total	including		
		Lectures	Practical	SRC
1	*2*	*3*	*4*	*5*
Section 1. *Series*				
Topic 1. *Series*	30	14	14	2
Topic 2. *Series and Fourier Integrals*	22	10	10	2
Total for Section 1	**52**	**24**	**24**	**4**
Section 2. *Functions of complex variables and their applications*				
Topic 3. *Functions of a complex variable*	38	18	16	4
Topic 4. *Operational calculus*	28	12	12	4
Total for Section 2	**66**	**30**	**28**	**8**
Calculation work	10	–	–	10
Module test	6	-	2	4
Exam	16	-	–	16
Total hours	**150**	**54**	**54**	**42**

1. **Teaching materials and resources**

Basic literature

2. **Dubovyk, V. P. Higher Mathematics / V. P. Dubovyk, I. I. Yurik. — Kyiv: Ignatex-Ukraine, 2013. — 648 p. http://library.kpi.ua:8991/F/V467KL684MQGAPRA4I9MDIFGD2VHBNMNQBARSIJGRU6SKI**
P181-01757?func=full-set-set&set_number=797795&set_entry=000003&format=999

3. Sequences. Functions of a complex variable. Operational calculus. Lecture notes. (2nd year, 1st semester) / Compiled by: V. O. Haidei, L. B. Fedorova, I. V. Alekseeva, O. O. Dykhovichnyi, — Kyiv: NTUU "KPI", 2013. — 108 p.
http://matan.kpi.ua/public/files/Konspekt%20Riady.%20FKZ.%20Operacijne%20chyslenia.pdf

4. Rows. Theory of complex variable functions. Operational calculus. Practical exercises. (2nd year, 3rd semester) / Compiled by: I. V. Alekseeva, V. O. Gaidei, O. O. Dykhovichnyi, L. B. Fedorova. — Kyiv: NTUU "KPI", 2012. — 160 p.
https://ela.kpi.ua/handle/123456789/16627

5. Dubovyk V. P. Higher Mathematics. Collection of Problems: Textbook. / V. P. Dubovyk, I. I. Yurik. — Kyiv: A.S.K., 2005. — 648 p.
http://library.kpi.ua:8991/F/V467KL684MQGAPRA4I9MDIFGD2VHBNMNQBARSIJGRU6SKI
P181-02049?func=full-set-set&set_number=797796&set_entry=000018&format=999

Supplementary literature

1. Gorlenko, S. V. Series. Theory of Complex Variable Functions. Operational Calculus: Collection of Problems for Typical Calculations / S. V. Gorlenko, L. B. Fedorova, V. O. Gaidey. – Kyiv: IVC Publishing House Polytechnika, 2003. – 36 p. http://matan.kpi.ua/public/files/%D0%A0%D1%8F%D0%B4%D0%B8.pdf

Information resources Distance learning courses:

1. Higher Mathematics. Mathematical Analysis 3. Dykhovychny O. O., Pavlenkov V. V., Kruglova N. V., Butsenko Yu. P.
https://do.ipo.kpi.ua/course/view.php?id=6976

Educational content

1. Methodology for mastering the academic discipline (educational component) Full-time/distance learning

Lectures

No No	Lecture topic and list of key questions (list of teaching aids, references to literature, and assignments for independent study)
1	Numerical series: general information. Basic concepts, convergence tests for geometric series. Properties of convergent numerical series, necessary condition for convergence, numerical series with positive terms and comparison theorems. *Recommended literature*: [1], pp. 493-498; [2], pp. 5-12.
2	Signs of convergence of numerical series. Signs of convergence of numerical series with positive terms: D'Alembert's, radical and integral signs of Cauchy, study of convergence of generalized harmonic series. *Recommended reading*: [1], pp. 498-510, [2], pp. 12-16.
3	Alternating series: definition, concepts of absolute and conditional convergence, properties of absolutely convergent series, Riemann's theorem (without proof). Alternating numerical series, Leibniz's theorem, estimation of the remainder of such a series. *Recommended reading*: [1], pp. 498-510, [2], pp. 16-22

4	Functional series. Basic concepts (points of convergence, regions of convergence, uniform convergence). Weierstrass's criterion for uniform convergence. Theorems on the continuity of the sum, term-by-term integration and differentiation of a functional series. *Recommended reading*: [1], pp. 512-516, [2], pp. 22-25.
5	Power series. Power series on the real axis and in the complex plane, Abel's first theorem, the concepts of radius, interval (circle) and region of convergence of a power series, formulas for the radius of convergence. Theorem on uniform convergence of a power series (Abel's second theorem), continuity of the sum of a power series, invariance of its radius. Convergence when integrating and differentiating term by term. *Recommended reading*: [1], pp. 516-521, [2], pp. 25-29.
6	Expansion of a function into a power series. Taylor series. Taylor's formula, the remainder term of Taylor's formula in Lagrange form (review of material from the first semester). Formulation of the problem of developing a function into a power series on a certain interval, the theorem on the uniqueness of power series development, the concept of Taylor and Maclaurin series. Conditions for representing a function by a power series. Development of some elementary functions into power series. *Recommended reading*: [1], pp. 521-527, [2], pp. 29-36.
7	Application of power series to finding values of integrals and solutions to differential equations. *Recommended reading*: [1], pp. 527-531, [2], pp. 36-37.
8	Expansion of a function into a trigonometric series. Fourier series. The concept of orthogonal and orthonormal systems of functions, trigonometric system of functions. Formulation of the problem of developing a function into a trigonometric series on a given interval, necessary condition for such a development, theorem on the uniqueness of such a development, concept of a trigonometric Fourier series. Sufficient conditions for developing functions into a trigonometric series (Dirichlet's theorem without proof). *Recommended reading*: [1], pp. 538-549, [2], pp. 36-44.
9	The form of the Fourier series and its coefficients for 2π -periodic and 2l-periodic functions defined on a symmetric interval, the form of the Fourier series for even and odd functions. Fourier series expansion of functions defined on an arbitrary interval $[a;b]$: the form of the Fourier series and formulas for its coefficients. *Recommended reading*: [1], pp. 549-551, [2], pp. 44-49
10	Complex form of the Fourier series: form of the series and formulas for its coefficients. Amplitude and phase spectra of the Fourier series. *Recommended reading*: [1], pp. 551–553, [2], pp. 49–53.
11	Development of of in series Fourier by orthogonal system functions *Recommended reading*: [1], pp. 553-557,
12	Fourier integral. Fourier integral: sufficient conditions for representing a function by a Fourier integral
	(formulation of Fourier's theorem), Fourier integral for even and odd functions. Complex form of the Fourier integral, concept of Fourier transform, sine and cosine Fourier transforms. Concept of spectral characteristics, amplitude-frequency and phase-frequency spectra. *Recommended reading*: [1], pp. 557-564, [2], pp. 89-93.

13	Functions of a complex variable: general information and basic elementary functions. Complex numbers (independent work on reviewing the topic from the 1st semester), complex plane, finite and extended complex plane, stereographic projection. The concepts of a domain and a closed domain, a simply connected domain and a multiply connected domain. The concept of a function of a complex variable, its limit, continuity, properties of continuous functions. Definition of basic elementary functions of a complex variable and their properties. Euler's formula. The relationship between hyperbolic and trigonometric functions. Calculation of the values of basic elementary functions of a complex variable. *Recommended reading*: [2], pp. 53-62.
14	The concept of a derivative of a function of a complex variable. Analytic functions. The concept of a derivative of a function of a complex variable, an analytic function, the Cauchy–Riemann (D'Alembert–Euler) conditions. The geometric meaning of the modulus and argument of a derivative. Conjugate harmonic functions. Finding an analytic function from one of its parts. *Recommended reading*: [2], pp. 62-68.
15	Integration of functions of a complex variable. Integral of a function of a complex variable: definition and properties. Cauchy's integral theorem. The concept of an indefinite integral, Newton–Leibniz formula. Cauchy's integral formula. Cauchy-type integral, theorem on its analyticity, existence of derivatives of any order from an analytic function, Morer's theorem. *Recommended reading*: [2], pp. 68–75.
16	Expansion of an analytic function into a power series. Expansion of an analytic function in a circle into a power series, the concept of a holomorphic function and its equivalence to the concept of a single-valued analytic function, the concepts of regular and singular points. *Recommended reading*: [2], pp. 75-80.
17	Laurent series. Expansion of an analytic function in a circular ring into a Laurent series, regular and principal parts of a Laurent expansion. *Recommended reading*: [2], pp. 80-83.
18	lessonification of special isolated points of unambiguous character. Laurent expansion in the neighborhood of an infinite point as special, lessonification of an infinite point as special. *Recommended reading*: [2], pp. 83-86.
19	The theory of residues. The concept of a residue, the fundamental theorem on residues. Finding residues, generalization of the fundamental theorem on residues. *Recommended reading*: [2], pp. 86-87.
20	Application of residue theory to the calculation of certain types of integrals of real functions. *Recommended reading*: [2], pp. 87-88.
22	Laplace transform. Definition of the original and image. Theorem on the domain of existence and analyticity of the image. Concept of Laplace transform, finding the image of the unit (Heaviside function) and exponential originals. Necessary property of the image. Properties of the Laplace transform: homogeneity, additivity, linearity. *Recommended reading:* [2], pp. 94-97.
23	Basic properties of the Laplace transform. Properties of the Laplace transform: similarity theorem, image of a periodic original, theorems on differentiation of the original and image, delay theorem, shift theorem, theorems on integration of the original and image. Convolution of originals: definition, simplest properties, and Borel's theorem on its image, Duhamel's formulas, table of simplest images. *Recommended reading*: [2], pp. 97–103.
24	Inverse Laplace transform. Finding the original for a fractional-rational image by decomposing it into the simplest rational fractions. Riemann–Mellin inversion formula, finding the original using residue theory and the Duhamel formula. *Recommended reading*: [2], pp. 103-105.

25	Application of calculus. Examples of operational calculus solving differential equations. *Recommended reading*: [2], pp. 105-107.
26-27	Application operational calculus. Examples of operational calculus to solving systems of differential equations and integral equations *Recommended reading*: [2], pp. 105-107.

Practical lessons

No No	Name of the topic and list of main questions (list of teaching aids, references to literature, and assignments for independent study)
1	Study of the convergence of numerical series by definition and comparison theorems. *Assignments for independent study*: [3], 1.5-1.9 (even numbers).
2	Research on the convergence of numerical series based on "nominal" characteristics. *Assignments for independent study*: [3], 2.8-2.14 (even numbers).
3	Study of the convergence of sign-alternating numerical series. Absolute and conditional convergence. Study of the absolute and conditional convergence of sign-alternating numerical series. *Assignments for independent work*: [3], 3.3-3.5 (even numbers);
4	Finding the convergence region of functional series. Investigation of their uniform convergence using the Weierstrass criterion. Application of uniform convergence. *Assignments for independent work*: [3], 4.6-4.10 (even numbers);
5	Finding the radius, interval (circle), and region of convergence of a power series. *Assignments for independent work:* [3], 5.3-5.5 (even numbers);
6	Techniques for developing functions into power series. Compiling a table of basic schedules. *Assignments for independent study*: [3], 6.5 (even numbers);
7	Application of developing functions in power series: approximate calculation of function values, definite integrals; approximate analytical solution of Cauchy's problem for differential equations, finding limits of functions. *Assignments for independent work*: [3], 6.6-6.10 (even numbers);
8	Fourier series expansion of 2π - and 2l-periodic functions defined on a symmetric interval. Fourier series expansion of non-periodic functions defined on an arbitrary interval. *Assignments for independent study*: [3], 7.8-7.10 (even numbers); [3], 7.11-7.14;
10	Complex form of the Fourier series. *Assignments for independent study*: 8.4 (even numbers);
11-12	Image of a function by Fourier. Fourier transform Fourier. Finding amplitude-frequency and phase-frequency characteristics. *Assignments for independent work*: [3], 16.3-16.5 (even numbers).
13	Operations on complex numbers. *Assignments for independent work*: [3], 9.5-9.7 (even numbers).
14	Finding the values of basic elementary functions of a complex argument. *Assignments for independent study*: [3], 9.8-9.8 (even numbers).
15	Derivative of a complex variable function. Investigating functions for monogenicity and analyticity. Finding an analytic function from one of its parts. Geometric meaning of a derivative. *Assignments for independent work:* [3], 10.6-10.11 (even numbers).

16	Integral of a complex variable function: finding integrals of non-analytic and analytic functions. *Assignments for independent work:* [3], 11.7-11.8 (even numbers).
17	Cauchy's integral formula. *Assignments for independent work*: [3], 11.9-11.12 (even numbers).
18	Methods for developing an analytic function into a power series. Finding the zeros of an analytic function and their multiplicity. Finding the special points of an analytic function and determining their nature. *Assignments for independent study*: [3], 12.3-12.6, 13.4-13.6 (even numbers).
19-20	Calculation of integrals of complex and some real functions using residue theory. *Assignments for independent work*: [3], 14.3-14.4, 15.3-15.6 (even numbers).
21	Finding images for given originals. Assignments for independent work: [3], 17.13-17.16 (even numbers).
23	Finding images of originals for given images. *Assignment for independent study:* [3], 18.5 (even numbers).
24	Finding originals for given images using the development theorem and Duhamel's formula *Assignments for independent work*: [3], 18.6-18.7 (even numbers).
25-26	Application of operational calculus. Examples of applying operational calculus to solving differential and integral equations and their systems. *Assignments for independent work*: [3], 19.7-19.11 (even numbers).
27	MODULE TEST

2. Independent work of a student/graduate student

The study of the discipline includes the following types of independent work:

- preparation for lectures and practical lessons, completion of homework assignments;
- completing computational work (test assignments in distance learning courses on the Moodle platform);
- preparation and completion of modular control work;
- preparation for the exam.

Control works

One Module test is planned for all topics. The purpose of Module test s is to determine the level of mastery of the relevant modules and to calculate points according to the credit-modular system of modules.

Policy and control

3. Policy of the academic discipline (educational component)

Recommended teaching methods: studying the main and supplementary literature on the topics of the lectures, solving problems in practical lessons and when doing homework.
Students are advised to take detailed lecture notes. An important aspect of high-quality mastery of the material and practice of methods and algorithms for solving the main tasks of the discipline is independent work. It includes reading literature, reviewing literature on the topic, preparing for lessons, performing computational work, and preparing for the Module test and exam.

Academic integrity

The policy and principles of academic integrity are defined in Section 3 of the Code of Honor of the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute." For more information, see: https://kpi.ua/code

Standards of ethical conduct

The standards of ethical conduct for students and employees are defined in Section 2 of the Code of Honor of the National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute." For more details, please visit: https://kpi.ua/code

4. Types of control and rating system for assessing learning outcomes (RSA) (full-time/distance learning)

Distribution of study time by type of lesson and assignment in accordance with the working curriculum.

Semester	Study time		Distribution of teaching hours				Control measures		
	Credits	academic hours	Lectures	Practical	Lab work	SRS + Ex.	MODULE TEST	RR	Semester assessment
1	5	150	54	54	-	42	1	1	exam

A student's grade for a course consists of the points they receive for

1) answers in practical lessons;
2) one test (the Module test can be divided into several tests);
3) one RR
4) answers on the exam.

The rating scale is R = 100 points. The starting scale is R_C = 50 points. The exam scale is R_E = 50 points.

Rating (weighting) point system and assessment criteria

1. Work in practical lessons

Weighted score –15. The maximum number of points for all practical lessons is 15 points.

0.0 – refusal to answer, lack of knowledge of the necessary theoretical material;

0.25 – knowledge of individual fragments of theoretical material, ability to apply some of them;.

0.5 – superficial knowledge of theoretical material, solving problems with the help of the instructor;

0.75 – good knowledge of theoretical material, ability to apply it;

1 – thorough knowledge of theoretical material, almost independent problem solving

2. Module test

Weighting score -20. The maximum number of points for all tests is 20 points.

Module test assessment criteria:

absence from the test – 0 points,

Module test assessment (in points) equals the percentage (of the maximum number of points, 20) of its completion.

If < 60% is completed, the test is not counted.

3. Course work (CW).

Weighted score – 15. CW assessment criteria:

Failure to complete RR – 0 points. RR is completed and defended in parts that correspond in content to the Module test . This part of RR is submitted before writing the MODULE TEST , and the MODULE TEST itself is its defense.

The RR assessment (in points) is equal to the percentage (of the maximum number of points, 15) of its completion, taking into account the result of writing the corresponding Module test.

If less than 60% of the RR is completed, it is not credited.

For late submission (more than a week late) of the RR, no more than 60% is counted.

4. Answer on the exam

Weighting score – 50.

The number of rating exam points is equal to the percentage (of the maximum score of 50) of the exam completion. If less than 60% (<30 points) of the exam is completed, it is not and must be retaken.

Bonus points are awarded for successful performance in the mathematics Olympiad (maximum 5 points per semester).

Conditions for a positive interim assessment.

To receive a "pass" on the first interim assessment (week 8), students must have at least 50% of the planned number of points. To receive a "pass" on the second interim assessment (week 14 week), the student must also have at least 50% of the planned number of points.

If a student is unable to write a Module test for valid reasons, they are given the opportunity to rewrite it within the next two weeks.

Retaking a positive final semester assessment in order to improve it is not permitted.

A student is admitted to the exam if their semester rating is not less than 30 points, and they have at least one positive assessment, credited Module test s, and a typical calculation (completed not less than 60%).

If the semester rating is less than 30 points but greater than 20, the student may write a make-up test. If the test is passed (at least 60% of the problems solved correctly), the semester rating will be 30 points.

Table for converting the rating assessment for an academic discipline R: *(according to Table 1)*

$R = R_I + R_E$	**ECTS grade**	**Traditional grade**
95	A	Excellent
85	B	very good
75	C	Good
65...74	D	satisfactory
60...64	E	sufficient
$R \leq 60$	Fx	unsatisfactory
$R_I < 30$ or other conditions for admission to the exam are not met for admission to the exam	F	not admitted

5. Additional information on the discipline (educational component)

During the legal regime of martial law, the educational process at Igor Sikorsky KPI for full-time and part-time higher education students is conducted remotely. In the case of distance learning, the educational process is organized using e-mail, Telegram, video conferences in Zoom, and the Moodle educational platform. Current control can be carried out in the form of test control works in Moodle. The RSA may also be changed in accordance with the order of the KPI and the decision of the department.

Work program for the academic discipline (syllabus): Compiled by:

Associate Professor of the Department of Mathematics and Physics, Candidate of Physical and Mathematical Sciences, Associate Professor O.O. Dykhovychnyi.

Approved by the Department of Mathematics and Physics (Minutes No. 13 of 11.06.2025).

Approved by the Methodological Council of the RTF (Minutes No. 6 of 26.06.2025).